

C. U. SHAH UNIVERSITY

Winter Examination-2019

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT3

Branch: Diploma(All)

Semester : 2

Date : 12/09/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{5\theta} = \underline{\hspace{2cm}}$
(A) 1 (B) 0 (C) 1/5 (D) 5
- b) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \underline{\hspace{2cm}}$
(A) e^2 (B) e (C) $e^{1/2}$ (D) None of these
- c) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \underline{\hspace{2cm}}$
(A) -1 (B) 1 (C) 0 (D) 2
- d) $\frac{d(k)}{dx} = \underline{\hspace{2cm}}$ (Where k is constant)
(A) -1 (B) 0 (C) 1 (D) None of these
- e) $\frac{d(\log x)}{dx} = \underline{\hspace{2cm}}$
(A) $-\frac{1}{x^2}$ (B) $\frac{1}{x}$ (C) e (D) 1
- f) $\frac{d(e^{ax})}{dx} = \underline{\hspace{2cm}}$
(A) ae^{ax} (B) e^{ax} (C) $\frac{e^{ax}}{a}$ (D) None of these
- g) $\frac{d(\cos^{-1} x)}{dx} = \underline{\hspace{2cm}}$
(A) $\frac{-1}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$
- h) $\int x dx = \underline{\hspace{2cm}}$



(A) 0 (B) 1 (C) $\frac{x^2}{2} + c$ (D) None of these

i) $\int 7^x dx = \underline{\hspace{2cm}}$

(A) $\frac{7^x}{\log_e 7} + c$ (B) $7^x \log_e 7 + c$ (C) $7^x + c$ (D) $\log_e 7 + c$

j) $\int_0^1 \frac{1}{1+x^2} dx = \underline{\hspace{2cm}}$

(A) π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) None of these

k) $-3(5, -1, -2) + 8(1, 1, 0) = \underline{\hspace{2cm}}$

(A) (7, 11, 6) (B) (7, -11, 6) (C) (7, 11, -6) (D) (-7, 11, 6)

l) If $\bar{a} = i + j - k$ then $\hat{a} = \underline{\hspace{2cm}}$

(A) $\frac{(1, -1, 1)}{\sqrt{3}}$ (B) $\frac{(-1, 1, 1)}{\sqrt{3}}$ (C) $\frac{(1, 1, -1)}{\sqrt{3}}$ (D) $\sqrt{3}$

m) If $x(3, 1) + y(4, 2) = (1, 0)$ then $x = \underline{\hspace{2cm}}$ and $y = \underline{\hspace{2cm}}$.

(A) 1, 2 (B) 2, 1 (C) $1, \frac{1}{2}$ (D) $1, -\frac{1}{2}$

n) If $\bar{x} = (2, 2, 1)$ and $\bar{y} = (1, -1, 1)$ then $\bar{x} \cdot \bar{y} = \underline{\hspace{2cm}}$

(A) 5 (B) 1 (C) 0 (D) -1

Attempt any four questions from Q-2 to Q-8

Q-2

Attempt all questions

(14)

a) If $\bar{a} = 2i + j - k$, $\bar{b} = i - j + 2k$ and $\bar{c} = i - 2j + k$ then find the direction cosines of $\bar{a} + \bar{b} - 2\bar{c}$. **(5)**

b) Find unit vector which is perpendicular to $\bar{a} = i + j + k$ and $\bar{b} = 2i - 2j + k$. **(5)**

c) Evaluate: $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + \dots + n^3}{n^2(1 + 2 + \dots + n)}$ **(4)**

Q-3

Attempt all questions

(14)

a) Prove that $\lim_{x \rightarrow \infty} \left[\sqrt{x} (\sqrt{x+p} - \sqrt{x}) \right] = \frac{p}{2}$ **(5)**

b) Evaluate: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \sec^2 x}{1 - \tan x}$ **(5)**

c) Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{ax+b}{cx+d}}$. **(4)**

Q-4

Attempt all questions

(14)

a) The equation of motion of a particle is $S = 2t^3 + 3t^2 - 12t + 5$. **(5)**

(i) Find velocity at $t = 0$. (ii) Find acceleration at $t = 1$.



b) Find $\frac{dy}{dx}$ if $y = \log \sqrt{\frac{a+x}{a-x}}$. (5)

c) If $\bar{a} = 2i - j$, $\bar{b} = i + 3j - 2k$ then obtain $\left|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})\right|$. (4)

Q-5

Attempt all questions

(14)

a) Prove that angle between two vectors $i + j - k$ and $2i - 2j + k$ is

(5)

$$\sin^{-1}\left(\frac{\sqrt{26}}{3\sqrt{3}}\right).$$

b) Find $\frac{dy}{dx}$ if $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$. (5)

c) Evaluate: $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^4 - 16}$ (4)

Q-6

Attempt all questions

(14)

a) Constant forces $3i - j + 2k$ and $i + 3j - k$ act on a particle and the particle moves from the point $2i + 3j + k$ to the point $5i + 2j + 3k$. Find the work done by the forces.

(5)

b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx = \frac{\pi}{4}$. (5)

c) Find derivative of $y = 3^{4x}$ using first principle. (4)

Q-7

Attempt all questions

(14)

a) Evaluate: $\int \left[\sqrt{1 + \sin 2x} + \sqrt{\frac{1 + \cos 2x}{1 - \cos 2x}} \right] dx$ (5)

b) Find the area of the region bounded between curve $y = x^2$ and straight – line $x = 2$. (5)

c) For what value of R , vectors $2i - 3j + 5k$ and $Ri - 6j - 8k$ are perpendicular to each other? (4)

Q-8

Attempt all questions

(14)

a) If $\frac{dy}{dx} = 4x^2 + 6x - 1$ and $y = 5$ when $x = 2$, represent y as a function of x . (5)

b) Find $\frac{dy}{dx}$ if $y = (\sin x)^{\tan x}$. (5)

c) Evaluate: $\int_0^{10} W dx$ Where $W = \frac{3}{4}x \left(1 + \frac{x}{10}\right)$ (4)

